Spatial patterns of basal drag inferred using control methods from three ice flow models for Pine Island Glacier

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Objectives

Basal friction remains a poorly understood aspect of ice dynamics. This parameter is crucial to better understand and model glacier evolution. Here, we use ice surface velocities of Pine Island Glacier from year 1996 derived from SAR interferometry to infer basal drag using an inverse control method on fully coupled thermomechanical models (i.e. for each iteration, a thermomechanical equilibrium is sought). We apply these inverse control methods on three different ice flow models corresponding to different levels of approximation of ice flow equations while keeping the same friction law given by Paterson[5]:

$$\vec{\tau_b} = -k^2 N_{eff}^r \|\vec{u_b}\|^{s-1}\vec{u_b} \qquad (1)$$

- $\vec{u_b}$ is the velocity component parallel to the bedrock surface
- N_{eff} is the effective pressure at the base
- $\vec{\tau_b}$ is the friction stress component parallel to the bedrock surface
- k, r and s are constants

We then compare the resulting basal drag patterns given by the three models to recommend the degree of precision required to properly model the glacier.
We used the following data:

- DEM from Bamber[1]
- Thicknesses from AGASEA[8]
- Velocities from Rignot 1996
- Surface temperature from Giovinetto[2]
- Geothermal flux from Maule[4]

Models

- Navier-Stokes: 3d incompressible (black, green and red)
- Pattyn/Blatter's higher order model[6] (black and green)
- MacAyeal's shelfy stream model[3] (black)

$$\begin{cases}
\frac{\partial}{\partial x}\left(2\mu\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y} + \mu\frac{\partial v}{\partial x}\right) + \frac{\partial}{\partial z}\left(\mu\frac{\partial u}{\partial z} + \mu\frac{\partial w}{\partial x}\right) - \frac{\partial P}{\partial x} = 0 \\
\frac{\partial}{\partial x}\left(\mu\frac{\partial u}{\partial y} + \mu\frac{\partial v}{\partial x}\right) + \frac{\partial}{\partial y}\left(2\mu\frac{\partial v}{\partial y}\right) + \frac{\partial}{\partial z}\left(\mu\frac{\partial v}{\partial z} + \mu\frac{\partial w}{\partial y}\right) - \frac{\partial P}{\partial y} = 0 \\
\frac{\partial}{\partial x}\left(\mu\frac{\partial u}{\partial z} + \mu\frac{\partial w}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu\frac{\partial v}{\partial z} + \mu\frac{\partial w}{\partial y}\right) + \frac{\partial}{\partial z}\left(2\mu\frac{\partial w}{\partial z}\right) - \frac{\partial P}{\partial z} - \rho g = 0 \\
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0
\end{cases}$$

The thermal model used in this experiment is:

$$\frac{\partial T}{\partial t} = -v\cdot\nabla T + \frac{k_{th}}{\rho c}\Delta T + \frac{\Phi}{\rho c} \qquad (2)$$

Anisotropic mesh generation

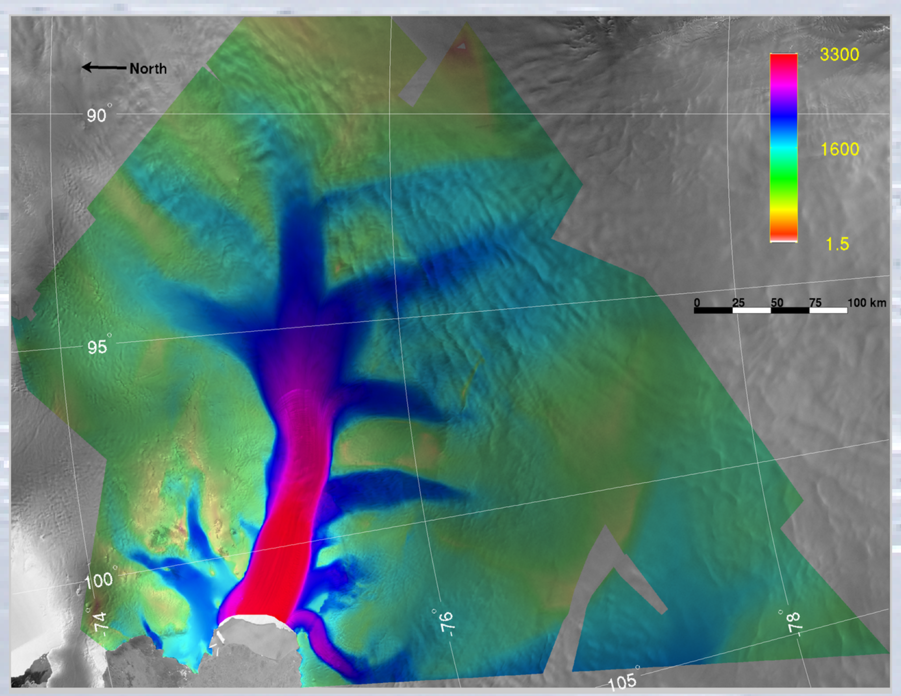
In any space of dimension d, if the solution $u(x)$ is approximated by $u^h(x)$, with piecewise linear interpolation, a local approximation error can be defined over an element E to be:

$$\text{error} = \left|u(x) - u^h(x)\right| \leq c_d h_E^2 \sup_{(x,y)\in E} |H_u(x,y)| \qquad (3)$$

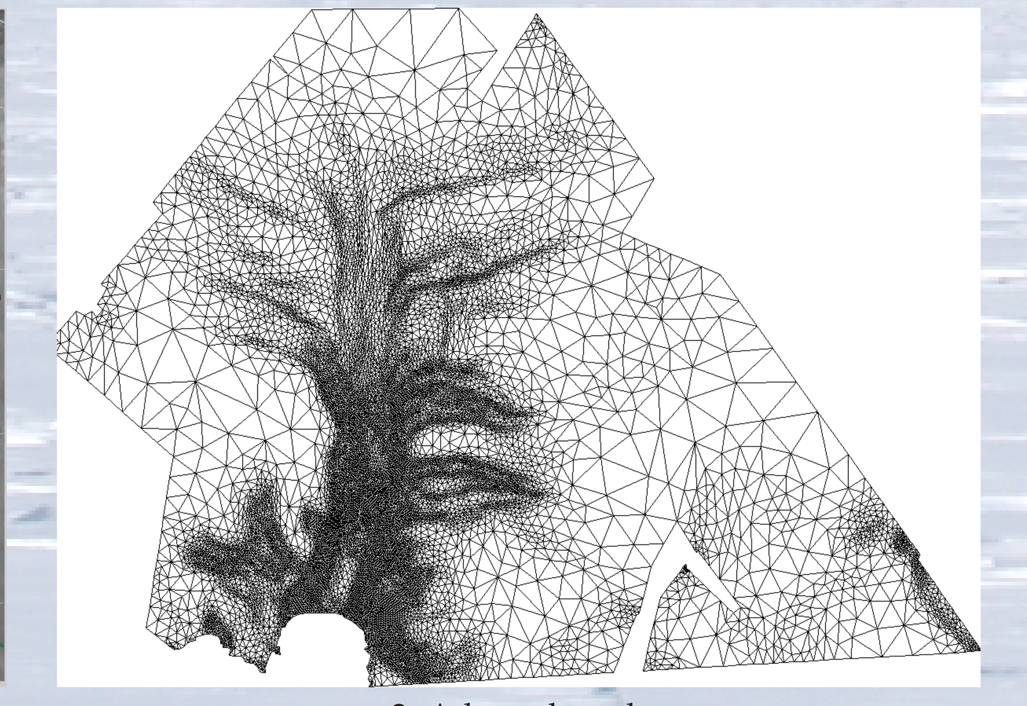
- h_E length of the element edge
- c_d constant that depends only on the space dimension (1/8 in 1d, 2/9 in 2d)
- $H_f(x,y)$ Hessian matrix of u, $|H_u(x,y)|$ its spectral norm

An optimized mesh can thus be defined as a mesh for which the maximum error estimate is equidistributed over all elements:

$$\forall E \qquad h_E^2 \sup_{(x,y)\in E} |H_u(x,y)| = Cte \qquad (4)$$

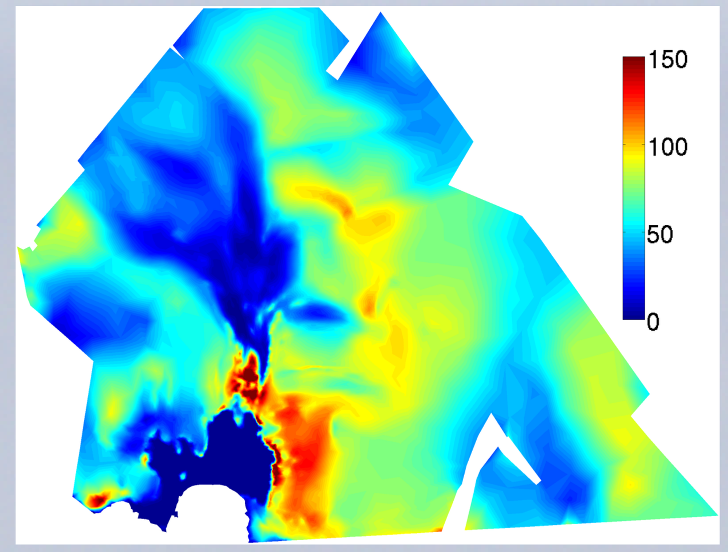


1. Observed velocity [m/yr]

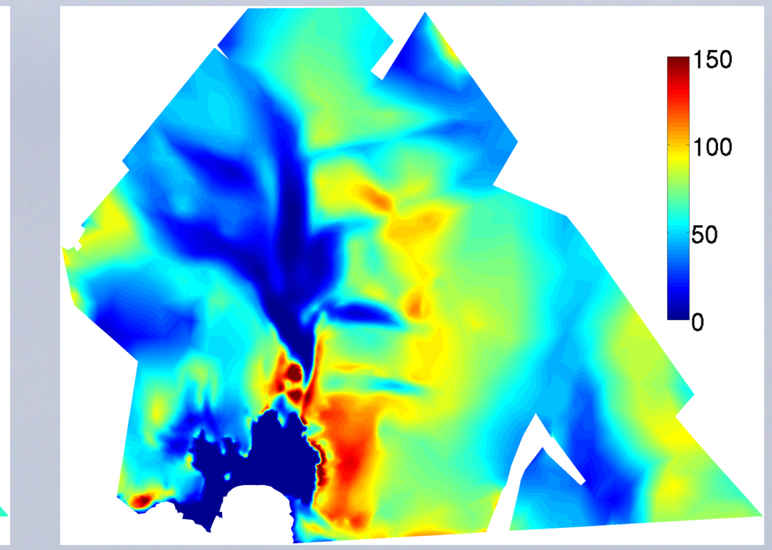


2. Adapted mesh

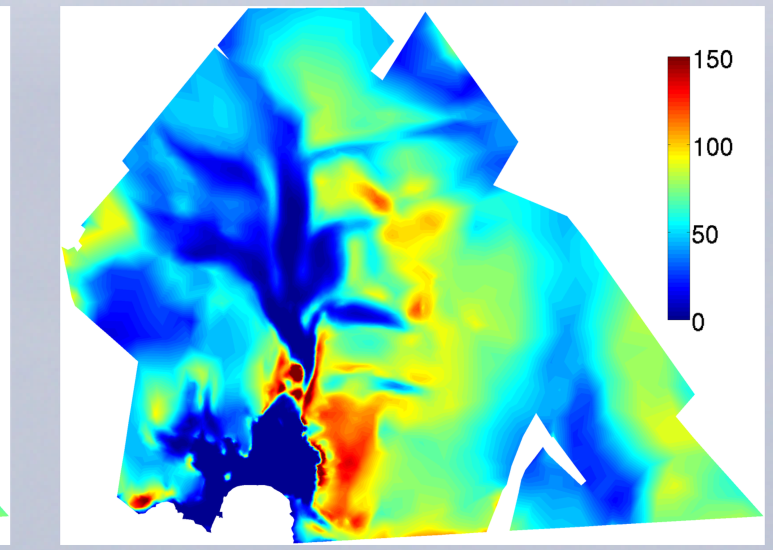
Experiment Results



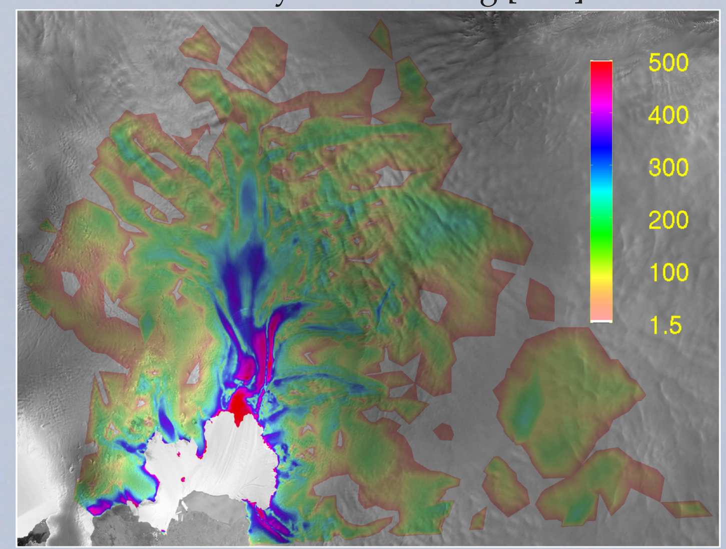
1. MacAyeal basal drag [kPa]



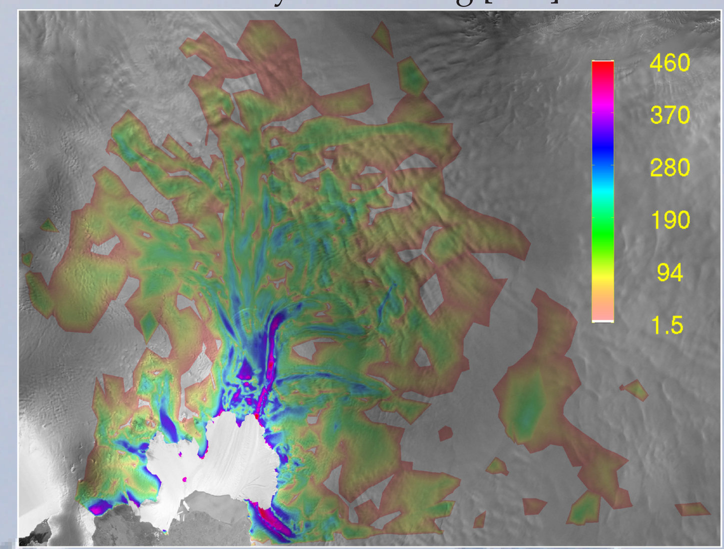
2. Pattyn basal drag [kPa]



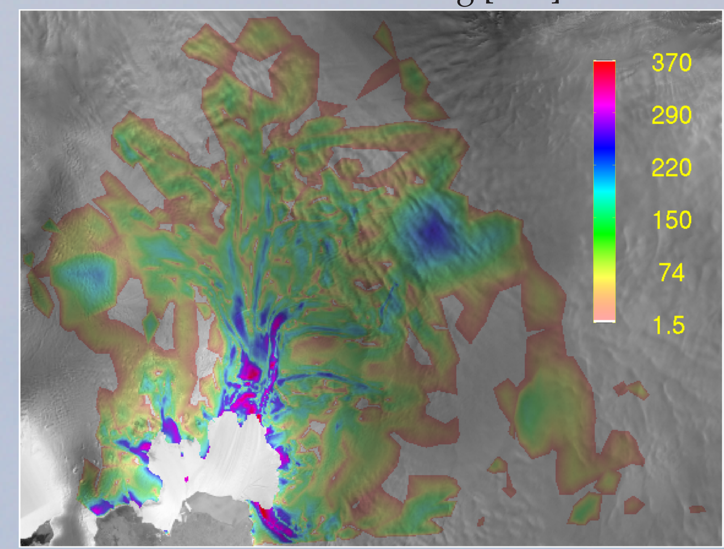
3. Stokes basal drag [kPa]



4. MacAyeal $\left|\text{vel} - \text{vel}^{\text{obs}}\right|$ [m/yr]



5. Pattyn $\left|\text{vel} - \text{vel}^{\text{obs}}\right|$ [m/yr]



6. Stokes $\left|\text{vel} - \text{vel}^{\text{obs}}\right|$ [m/yr]

1 Results

We observe that the patterns of basal drag of the three models are very similar. The differences between the observed and modeled velocities are mainly located on the ice stream close to the grounding line where the glacier speed reaches its maximum. Overall, the three models manage to reproduce the observations with good accuracy (the following table shows the misfits between the modeled velocities and the observations):

Model	Ice sheet	Ice stream
MacAyeal's misfit	27 m/yr	62 m/yr
Pattyn's misfit	11.1 m/yr	22.9 m/yr
Stokes's misfit	10.4 m/yr	19.5 m/yr

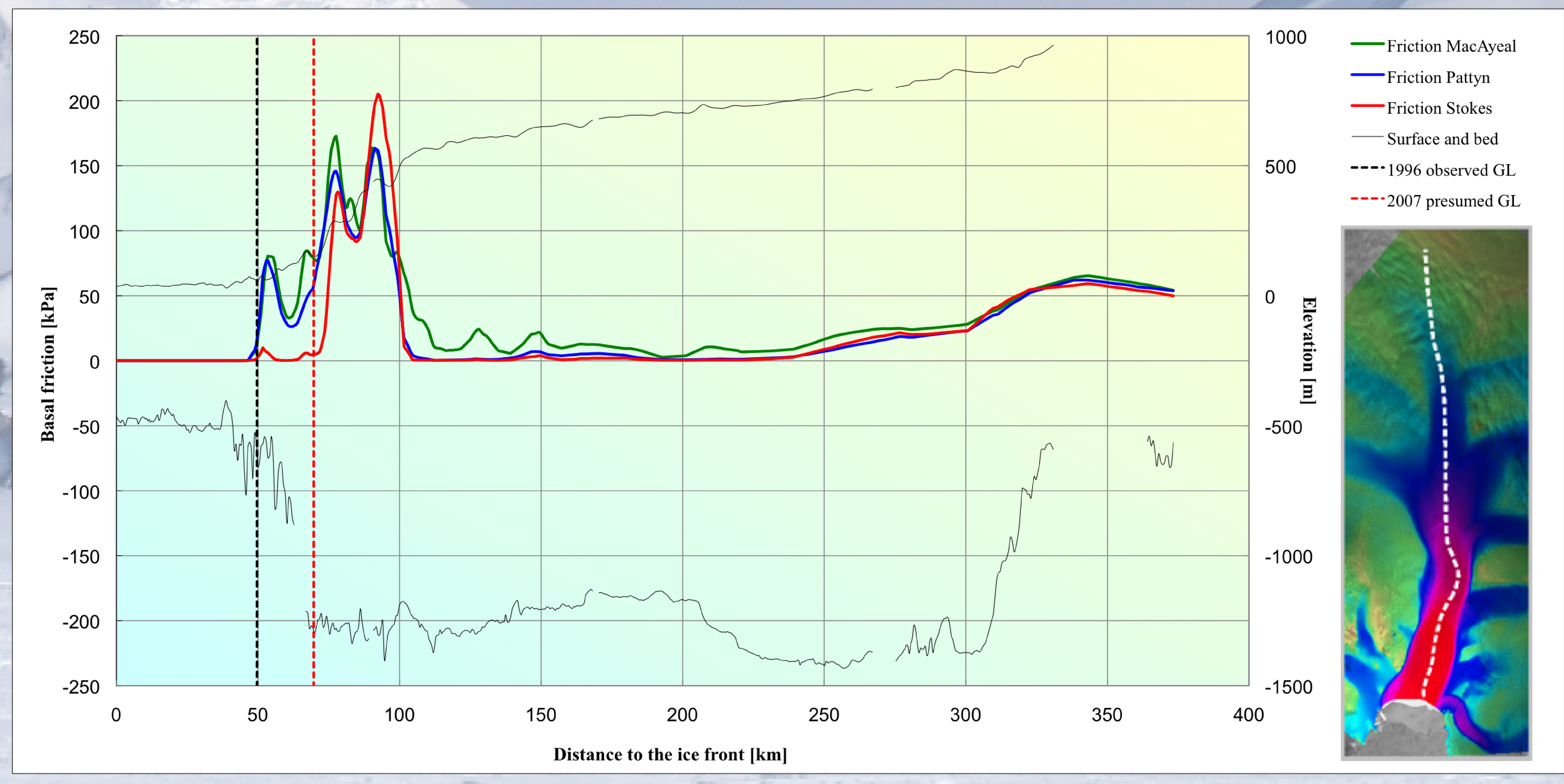
This means that the assumptions of no vertical shear made in MacAyeal's model remain valid almost everywhere and this model is a good approximation of the glacier flow on most of the domain.

2 Discussion

Nevertheless, close to the grounding line the bed becomes very steep (3%) and it is there that the differences in basal drag patterns between the models arise. We observe that Stokes's shows almost no friction whereas MacAyeal's and Pattyn's models require strong basal friction to reproduce the observations. This is due to the fact that in a 3d model, the steep bed is slowing down the ice flow by itself and this effect is not taken into account by the other models as the vertical equation of the momentum balance is reduced to its simpler form.
Interestingly, the basal drag given by Stokes is close to zero over the distance of the grounding line retreat[7].

3 Conclusion

The three models seem to reproduce well the observed velocities and we find good agreement in the basal drag patterns. Nevertheless, the results show that close to the grounding line 3d effects cannot be neglected as the bed becomes steeper, which invalidates the assumptions made in MacAyeal's shelfy stream model and Pattyn's higher order model.



7. Friction comparison on a flow line

Acknowledgement and references

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